TWO-LEVEL MODEL OF THE DYNAMIC DEFORMATION OF METALS

Yu. I. Meshcheryakov and G. G. Savenkov

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In order to describe plastic deformation of metals under shock waves kinetic relationships are suggested which determine at the first level the plastic strain rate in terms of the multiplication rate and displacement velocity of mobile dislocations in relation to the operating shear stress. At the second level, at which rotation modes play a governing role, the plastic strain rate is a function of the displacement velocity of the dipoles of partial dislocations. Examples are provided of elastoplastic wave fronts in steel and aluminum.

Works devoted to microstructural models of elastoplastic wave propagation with dynamic loading are based on dislocation–kinetic relationships including kinematic equations of the form [1-3]

$$\dot{\gamma} = b\dot{N}\lambda + bN_m v \quad \text{or} \quad \dot{\gamma} = bN_m v \tag{1}$$

($\dot{\gamma}$ is plastic shear strain rate, $\dot{N}$ is dislocation multiplication rate, b is Burgers vector, λ is average dislocation path length, N_m is mobile dislocation density, and v is their average velocity), and also a rule for dislocation movement in one form or another determining the dependence of dislocation velocity on applied shear stress.

However, for satisfactory agreement of calculated and experimental data for the evolution of a wave front as a rule it is necessary to adjust some or other parameters (mostly connected with dislocation density) which are in the above-named set of equations [1].

It is possible to understand the behavior of materials with shock-wave loading better by noting the considerable progress in recent years in understanding the microstructural aspects of large plastic strains which is based on introducing structural scale levels of plastic strain, its rotation modes, and collective properties of dislocations and disclinations [4-6]. It should be noted that collective dislocation effects develop on reaching a critical value for dislocation density [4], and the reasons for rotation effects are relaxation of internal force moments, the work of applied stress moments, dislocation instability, and material plastic property anisotropy [5].

It is shown in a number of works that plastic strain in its developed phase with shock-wave loading is accomplished in a form of movement for a series of microflows [7-9] which in essence are developed of current dislocation instability. Further misorientation of them [10] points to inclusion of a disclination (rotation) mechanism of plastic strain.

Thus, it is possible to suggest that plastic strain with dynamic loading is accomplished at two scale levels: in the first (microscopic) it is elementary carriers, i.e., individual dislocations, and in the second (mesoscopic) it is the dislocation current and disclination dipoles, quadrupoles, etc. [4, 5]. In accordance with this each of the levels has its own part in the elastoplastic wave front.

It is noted that probably a two-level model of elastoplastic deformation of metals in shock waves was first considered in [11] where in contrast to this work microscopic and macroscopic strain levels were mentioned, and the conservation equation together with a fundamental equation of the relaxation type was solved numerically.

In order to evaluate the reality of the model suggested by us we consider in more detail calculation of a space–time compression pulse for the unidimensional case. For this purpose we write a set of dynamic equations for a solid material

$$\rho_0 \frac{\partial u}{\partial t} = \frac{\partial \sigma}{\partial x}, \quad \frac{\partial \varepsilon}{\partial t} = \frac{\partial u}{\partial x} \tag{2}$$

with a fundamental equation in the form

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$$\frac{\partial\sigma}{\partial t} - (\lambda_1 + 2G)\frac{\partial\varepsilon}{\partial t} = -\frac{8}{3}G\dot{\gamma}. \tag{3}$$

Here σ is normal stress in the direction of wave propagation; ε is total (elastic plus plastic) strain; u is particle mass velocity; ρ_0 is material density; λ_1 is Lamé constant; G is shear modulus.

For the first (dislocation) level the solution of set of Eqs. (2) and (3) with fulfillment of a regenerative rule for multiplication of dislocations

$$N_m = N_{m0} + \alpha\gamma \tag{4}$$

(N_{m0} is initial density of mobile dislocations, α is dislocation multiplication factor) and taking account of the Gilman dependence of dislocation velocity on shear stress

$$v = c_t \exp(-(\tau_0 + H\gamma)/\tau) \tag{5}$$

(c_t is transverse shear wave velocity, τ_0 is stress reflecting the overall level of barriers in a dislocation path, H is strengthening factor taking account of processes of dislocation stoppering with a high density of them) is well-known [12, 13]:

$$\sigma_I = \frac{8}{3}\frac{N_{m0}}{\alpha}G - \rho_0 c^2\left(\left(\frac{\tau_* + B_*}{\delta}\right) + \frac{2N_{m0}}{\alpha}\right) + \frac{2N_{cr}}{\alpha}\rho_0 c_h^2 \exp(-k(x - c_p t)). \tag{6}$$

Here $\tau_* = \tau_0/G$; $B_* = H/G$; $N_{cr} = 10^{13}\text{-}10^{14}$ m^{-2} is critical dislocation denstiy; c is longitudinal sound wave velocity; c_h is hydrostatic sound velocity; $\delta = \ln(\theta/bc_t\alpha G)$; $\theta = bc_t\alpha \times \exp(B_*/M)$; $k = \theta/c_p$; $M = 2(c_h^2 - c_p^2)/(c^2 - c_p^2)$; c_p is plastic wave velocity.

In the second stage of plastic strain its rate is determined by the expression

$$\dot{\gamma}_{II} = Nbv + 2n\omega a v_d \tag{7}$$

[n is density of mobile dipoles of partial disclinations (DPD), ω, 2a, and v_d are dipole magnitude, arm, and velocity].

We find the DPD velocity by making two assumptions: 1) there is no multiplication of disclinations with movement of dipoles; 2) stress in an elastoplastic wave exceeds the critical stress starting from which collective dislocation reconstructions are possible.

It is evident that the second assumption for intense dynamic loads, which are realized in the elastoplastic waves in question, is almost always fulfilled. Then the DPD velocity is determined by the relationship [5]

$$v_d = \left(\int_{-a}^{a} v(y)\,dy\right)\Big/\lambda \ln(1 - \omega/bN_1\lambda) \tag{8}$$

(N_1 is initial dislocation density equal in our case to N_{cr}). Mobile dislocation velocity in the second part is determined from the rule for viscous retardation

$$v = (\tau - \tau_0)b/B \tag{9}$$

(B is dislocation viscous retardation factor).

By substituting (9) in (8) and assuming the $\lambda \simeq a$, we obtain

$$v_d = -\frac{2(\tau - \tau_0)b}{B\ln(1 - \omega/bN_{cr}a)}. \tag{10}$$

Dipole density is determined in the form $n = \beta N_{cr}$ (β = const $\ll 1$). According to estimates provided in [14], $n_{max} = 10^{11}$ m^{-2}, i.e., $\beta_{max} \simeq 10^{-3}\text{-}10^{-2}$.

By identifying plastic strain channels with partial disclination dipoles we have obtained by scanning electron microscopy the following values for a series of structural materials: for steels 12Kh18N10T, St. 4, and steel 45, $n = 9\cdot10^8$, $8\cdot10^9$, and $2\cdot10^9$ m^{-2}. These materials were tested for the effect of a shock wave with an intensity of about 50 GPa.

Taking account of (10) Eq. (7) takes the form

$$\dot{\gamma}_{II} = \frac{\tau - \tau_0}{B}N_{cr}b^2\left(1 - \frac{4\beta\omega a}{b\ln(1 - \omega/bN_{cr}a)}\right) \tag{11}$$

and (3) has the form

$$\dot{\sigma}_{II} - \rho_0 c^2\dot{\varepsilon}_{II} = -\frac{8}{3}GN_{cr}b^2\left(1 - \frac{4\beta\omega a}{b\ln(1 - \omega/bN_{cr}a)}\right)\left(\frac{\tau - \tau_0}{B}\right). \tag{12}$$

In terms of shear strain taking account of the fact that

$$\sigma - (\lambda_1 + 2G)\varepsilon = -\frac{8}{3}G\gamma; \tag{13}$$

$$\tau = \frac{3}{4}\left(\sigma - \left(\lambda_1 + \frac{2}{3}G\right)\varepsilon\right) \tag{14}$$

(τ is shear stress), Eq. (12) may be rewritten as

$$\dot{\gamma}_{\rm II} = \frac{G((\varepsilon - 2\gamma) - \tau_0) N_{\rm cr} b^2}{B}\left(1 - \frac{4\beta\omega a}{b \ln(1 - \omega/bN_{\rm cr} a)}\right). \tag{15}$$

In the case of a stationary plastic front system (2)-(5) takes the form

$$\rho_0 c_p \frac{\partial u}{\partial x} + \frac{\partial \sigma}{\partial x} = 0; \tag{16a}$$

$$c_p \frac{\partial \varepsilon}{\partial x} + \frac{\partial u}{\partial x} = 0; \tag{16b}$$

$$c_p \frac{\partial \gamma}{\partial x} = \frac{GN_{\rm cr}}{B}((\varepsilon - 2\gamma) - \tau_0)\left(1 - \frac{4\beta\omega a}{b \ln(1 - \omega/bN_{\rm cr} a)}\right). \tag{16c}$$

Integration of Eqs. (16a) and (16b) with respect to x gives

$$\rho_0 c_p u + \sigma = c_1,\ c_p \varepsilon + u = c_2, \tag{17}$$

and constants are found from conditions for the variables sought behind the region of a rapid change in them. Let ε_1, σ_1, and u_1 be values for strain, stress, and particle mass velocity at the end of the first part, and ε_0, σ_0, and u_0 be the maximum values of these quantities. Then system (17) is written as

$$\rho_0 c_p u + \sigma = \rho_0 c_p u_0 + \sigma_0,\ c_p \varepsilon + u = c_p \varepsilon_1 + u_1. \tag{18}$$

By substituting (13) in (18) we obtain

$$\rho_0 c_p u + \rho_0 c^2 \varepsilon - \frac{8}{3}G\gamma = \rho_0 c_p u_0 + \sigma_0,\quad c_p \varepsilon + u = c_p \varepsilon_1 + u_1. \tag{19}$$

From these equations it is possible to find stresses and strains in a plastic front in terms of plastic shear strain γ whose dependence on time and coordinate may be determined from the combined solution of Eqs. (15) and (16c), and as a result of this we have

$$\sigma_{\rm II} = \frac{1}{c_p^2 + c^2}\left(\rho_0 c^2\left(2c_p u_{\rm av} + c_p^2 \varepsilon_1 + \frac{\sigma_0}{\rho_0}\right) - \frac{8}{3}\Phi G c_p^2 + \frac{8}{3}c_p^2 G \exp(-k_1(x - c_p t))\right), \tag{20}$$

where

$$u_{\rm av} = \frac{u_1 + u_0}{2};\quad \Phi = \frac{2u_{\rm av} + c_p \varepsilon_1 - \tau_*(c_p^2 + c^2)}{4G - \rho_0(c_p^2 + c^2)}\frac{3}{2}\rho_0 c_p.$$

In this case angular frequency

$$\theta = \frac{2N_{\rm cr} G b^2}{B}\left(1 - \frac{4\beta\omega a}{b \ln(1 - \omega/bN_{\rm cr} a)}\right)\left(\frac{c_{\rm h}^2 + c_p^2}{c^2 + c_p^2}\right).$$

In accordance with expressions (6) and (20) the leading front of a compression pulse is in the form of an exponential curve growing due to stresses in the elastic precursor, i.e., the Hugoniot elastic limit (which has a point of inflection A in Figs. 1 and 2 with $\gamma = \gamma_{\rm cr}$), to the maximum stress value in the plateau of pulse σ_0.

Shown in Figs. 1 and 2 are a calculated (broken line) and an experimental (solid line) pulse for a target of steel 30KhN4M and aluminum grade A6. The time profile for free surface velocity was recorded by means of a laser differential interferometer. Values of material characteristics are given in Table 1. As can be seen from Figs. 1 and 2 quite good qualitative and quantitative agreement is observed for calculated and experimental results. Some small divergences may be explained by the error in treating experimental results, and also the approximate nature of the values for material dislocation–disclination structure characteristics.

However, it should be noted that in a number of works (see for example [1, 11, 15, 16]), in which good conformity of calculated and experimental data was also observed, points of inflection were not detected either by experiment or calculation. This situation suggests that the model proposed by us has limitations connected to all appearances with the original

TABLE 1

Material	v_0, m/sec	p_E, GPa	N_{m0}, m^{-2}	b, m	H,GPa	α, m^{-2}	ω, rad
30KhN4M	364	1.64	10^9	$2{,}48\cdot10^{-10}$	18	$2{,}4\cdot10^{11}$	0,06
A6	185	0,27	$1{,}5\cdot10^{11}$	$2{,}86\cdot10^{-10}$	7,2	$3\cdot10^{11}$	0,1

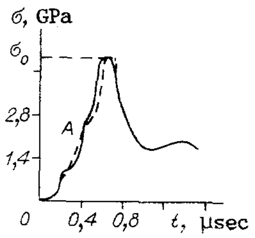

Fig. 1

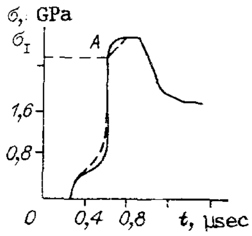

Fig. 2

parameters (N_{m0}, N) of the dislocation structure and the disclination structure formed (n and N_d, i.e., the number of disclinations in a DPD), and also with initial parameters of shock-wave loading.

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